AFRL-VA-WP-TP-2002-325

ROBUSTNESS STUDY OF THE DYNAMIC INVERSION BASED INDIRECT ADAPTIVE CONTROL OF FLIGHT VEHICLES WITH UNCERTAIN MODEL DATA

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OCTOBER 2002

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AIR VEHICLES DIRECTORATE
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AIR FORCE MATERIEL COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7542

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
October 2002	Conference Paper Preprint	

4. TITLE AND SUBTITLE
ROBUSTNESS STUDY OF THE DYNAMIC INVERSION BASED INDIRECT ADAPTIVE CONTROL OF FLIGHT VEHICLES WITH UNCERTAIN MODEL DATA

5a. CONTRACT NUMBER
F33615-01-2-3154

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
N/A

6. AUTHOR(S)
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5d. PROJECT NUMBER
N/A

5e. TASK NUMBER
N/A

5f. WORK UNIT NUMBER
N/A

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

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Air Vehicles Directorate
Air Force Research Laboratory, Air Force Materiel Command
Wright-Patterson Air Force Base, OH 45433-7542

8. PERFORMING ORGANIZATION REPORT NUMBER
ACC03-AIAA0020

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Vehicles Directorate
Air Force Research Laboratory
Air Force Materiel Command
Wright-Patterson Air Force Base, OH 45433-7542

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL/VACA

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-VA-WP-TP-2002-325

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
Proceedings for American Control Conference, June 4, 2003, Denver, CO.

14. ABSTRACT ***(Maximum 200 Words)***
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15. SUBJECT TERMS
Feedback Linearization, Dynamic Inversion, Indirect Adaptive Control, Uncertainties, Robustness

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
a. REPORT	b. ABSTRACT	c. THIS PAGE	SAR	20	David B. Doman
Unclassified	Unclassified	Unclassified			19b. TELEPHONE NUMBER *(Include Area Code)* (937) 255-8451

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18

ACC03-AIAA0020

Robustness Study of the Dynamic Inversion Based Indirect Adaptive Control of Flight Vehicles with Uncertain Model Data

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Abstract

The objective of this paper is to analyze the stability robustness of dynamic inversion based control laws being used for flight control with uncertainties in model data such as the aerodynamic stability and control derivatives. In particular, the proposed method is aimed at determining the robustness of an indirect adaptive control system developed by the researchers at the AFRL, which is built around a baseline dynamic inversion inner loop. MATLAB/SIMULINK is the environment used for the development and simulation of the flight controller. While this control law is quite successful in tracking the desired angular velocity commands, no explicit parametric stability robustness margins are provided. The controller, which is used to track the desired angular velocity commands, under perfect dynamic inversion, would result in a bank of integrators in the three angular velocity channels. But due to the existence of dynamic inversion errors resulting from uncertainties in the parameters such as the control derivatives, robustness of the designed control system needs to be guaranteed. In order to study the robustness of the controller, in this paper, an innovative method that takes advantage of the specific nature of the equations of motions of the aircraft is used to aid in the analysis of the closed loop system. Using this technique, point-wise controllability and stability are shown to be sufficient to guarantee the stability of the overall closed loop system. The method exploits the unique nature of the equations of motion of the aircraft to develop a closed loop system under dynamic inversion so that appropriate perturbation study can be carried out on the system. The nonlinear equations of motion are expressed in the form that is termed State Dependent Linear State Space System and this allows us to carry out robustness studies on the nonlinear system using linear system techniques.

Keywords: Feedback Linearization, Dynamic Inversion, Indirect Adaptive Control, Uncertainties, and Robustness.

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Nomenclature

B	=	Control Effectiveness matrix
BAE	=	Contributions from base aerodynamics and engine
C	=	Matrix relating ω and the state variables
δ	=	Control surface deflection vector
delta	=	Contributions from control surface deflections
G	=	Moment Vector
I	=	Inertia Tensor
L, M, N	=	Roll, pitch and yawing moments
MRP	=	Moment Reference Point
P, Q, R	=	Roll, pitch and yaw velocities
P	=	Parameter Vector
ϕ, θ, ψ	=	Euler angles of rotation
ω	=	Angular Velocities ([P; Q; R])

Introduction

The reentry vehicle considered in this study is representative of the X-40A Space Maneuvering Vehicle in that it has a similar shape with four control surfaces: right/left tails and right/left flaps. Such a vehicle would be carried to orbit on a reusable or expendable launch vehicle for on-orbit missions, while reentering the atmosphere at hypersonic speeds and finally landing horizontally like an airplane. Such a vehicle would have a very large flight envelope that spans a very wide range of speeds and altitude

In the method proposed by the researchers at AFRL [1], a nonlinear control law commands body-axis rotation rates that align the angular velocity vector with an Euler axis defining the axis of rotation that will rotate the body axis system into a desired axis system. The magnitudes of the commanded body rates are determined by the magnitude of the rotation error. The commanded body rates form the input to a dynamic inversion-based adaptive/reconfigurable control law. The baseline control law is based on a full envelope design philosophy and eliminates trajectory dependent gain scheduling that is typically found on this type of vehicle. The indirect adaptive control portion of the control law uses on-line system identification to estimate the current control effectiveness matrix to update a control allocation module.

Dynamic inversion control laws require the use of a control mixer or control surface allocation algorithm when the number of control surfaces exceeds the number of controlled variables. This is because a small number of desired moment or acceleration commands are calculated and a large number of control surfaces may be used to achieve the desired command. It is quite common that the desired commands can be achieved in many different ways and so control allocation approaches are used to provide consistent and unique solutions to such problems. The control allocation relies on accurate knowledge of the control derivatives. Under failure or damage conditions the control derivatives can be altered dramatically. Identifying control derivatives and supplying updated information to the control allocation block can improve the performance of the entire system. A fault-detection scheme is used to trigger an on-line system identification algorithm that is used the estimate control effectiveness when faults are detected.

In reference [2], a Linear Quadratic Gaussian outer-loop controller is developed that improves the robustness of a Dynamic Inversion inner-loop controller in the presence of uncertainties. The selected dynamics are based on both performance and stability robustness requirements. The method however is based the linearized equations of motion of the aircraft. An attempt is made in [3] to quantify the particular form of desired dynamics, which produce the best closed-loop performance and robustness in a Dynamic Inversion flight controller. Candidate forms of desired dynamics which invert the short period dynamics are evaluated. The controllers are synthesized for the prototype X-38 Crew Return Vehicle using a *linear model* at a selected point in the flight envelope.

This paper proposes a method that utilizes the unique nature of the equations of motion of the aircraft to develop a closed loop system under dynamic inversion so that an

appropriate perturbation study can be carried out on the system. The emphasis is to maintain the nonlinear nature of the indirect adaptive control law proposed in [1], and not rely on linearization and the resulting gain scheduling procedures, which is done in the current literature as mentioned above. When using the nonlinear indirect adaptive control law of [1], it is difficult to explicitly develop parametric stability margins using the resulting nonlinear closed loop system. However a close examination of the specific nature of the equations of motion for flight vehicles reveals that the nonlinear equations of motion $\dot{x}=f(x)+g(x,\delta)$ can actually be expressed in the form $\dot{x}=A(x)x+\mathbf{B}\delta$, with A(x) itself having a very desirable property described later in the paper. This special description of the equations of motion allows us to carry out robustness studies on the nonlinear system using linear system techniques. Thus the proposed method attempts to study the robustness of the control system across the entire flight envelope and not just at a selected point. In addition, the special nature of A(x) enables us to prove that point wise stability can guarantee the stability of the overall closed loop system.

Brief Overview of AFRL's Dynamic Inversion Control Law

Dynamic Inversion is a design technique used to synthesize flight controllers whereby the set of existing dynamics are cancelled out and replaced by a designer selected set of desired dynamics. The output of such an inner loop controller is the control input required to achieve the desired response. Dynamic inversion is similar to model following control, in that both methodologies invert dynamical equations of the plant. Whereas model-following control specifies the desired plant behavior with an internal model to be followed, dynamic inversion specifies the desired plant behavior explicitly by specifying the rate of the desired control variable, not the control variable itself.

A quaternion-based outer-loop control system designed for the ascent phase of the X-33 generated body-axis angular velocity commands *Pc, Qc, R*c that were aligned with the error Euler-axis [1]. The inner-loop dynamic inversion control law was designed so that the vehicle tracked these body-rate commands.
During the descent/landing flight phase, the reentry vehicle tracks the body rate commands generated by the guidance system. The rotational dynamics can be written as:

$$\dot{x}=f(\omega,P)+g(P,\delta) \quad (1)$$

where (ω = [p q r] and **P** denotes measurable or estimable quantities that influence the body- rate states. The parameter vector **P** includes variables such as Mach number, angle of attack, sideslip angle and vehicle *mass* properties such as moments of inertia. Equation (1) expresses the body-axis rotational accelerations as a sum that includes control dependent accelerations g(**P**, δ) and accelerations that are due only to the *base* engine and aerodynamics.

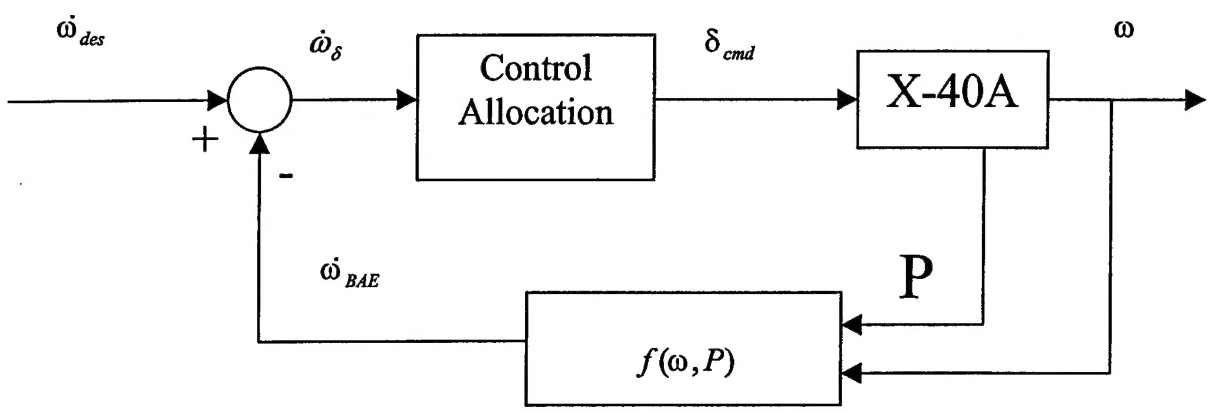

Figure 1

The angular accelerations of the aircraft are given by the following equation

$$\dot{\omega} = I^{-1}(G_B - \omega \times I\omega) \tag{2}$$

where

$$G_B = G_{BAE}(\omega,P) + G_\delta(P,\delta) = \begin{bmatrix} L \\ M \\ N \end{bmatrix}_{BAE} + \begin{bmatrix} L \\ M \\ N \end{bmatrix}_{\delta elta} \tag{3}$$

$G_{BAE}(\omega, P)$ is the moment generated by the base engine-aerodynamic system and G_δ is the sum of the moments produced by the control surfaces. Thus

$$f(\omega,P) = I^{-1}(G_{BAE}(\omega,P) - \omega \times I\omega) \tag{4}$$

and

$$g(P,\delta) = I^{-1}G_\delta(P,\delta) \tag{5}$$

Dynamic inversion requires that the control dependent portion of the model be affine in the controls. We therefore develop a linear approximation of the control dependent part

such that:

$$G_\delta(P,\delta) \approx G_\delta(P)\delta \tag{6}$$

The aerodynamic database for the re-entry vehicle under consideration provides force and moment coefficient data that is taken at a moment reference point (MRP) that is located at the center of gravity of the empty vehicle (i.e. no fuel/oxidizer). Control derivative information is extracted from the tables in the database for Mach numbers, angles of attack and sideslip angles that were to be encountered on the trajectory.

The model used for the design of the dynamic inversion control law becomes

$$\dot{\omega} = f(\omega,P) + G_\delta(P)\delta \tag{7}$$

and the control law provides direct control over $\dot{\omega}$ so that $\dot{\omega} = \dot{\omega}_{des}$ i.e.

$$\dot{\omega}_{des} = f(\omega,P) + G_\delta(P)\delta \tag{8}$$

Therefore, the inverse control law satisfies

$$\dot{\omega}_{des} - f(\omega,P) = G_\delta(P)\delta \tag{9}$$

Since there are more control surfaces than controlled variables, a control allocation algorithm must be used to obtain a unique solution. There are four control surfaces that may be used on descent: Right and Left Tails and Right and Left Flaps. Equation (9) states that the control surfaces are to be used to correct for the difference between the desired accelerations and the accelerations due only to the base engine and aerodynamic moments.

Special Nature of the Aircraft Equations of Motion: State Dependent Linear State Space System

Let us now consider the nonlinear equations of motion of the aircraft that are typically described in [6]. Consider only the six states including the rotational dynamics and the Euler angles. The six equations of motion are given by

$$\begin{aligned}
\dot{\phi} &= P + Q\sin\phi\tan\theta + R\cos\phi\tan\theta \\
\dot{\theta} &= Q\cos\phi - R\sin\phi \\
\dot{\psi} &= Q\sin\phi\sec\theta + R\cos\phi\sec\theta \\
\begin{bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \end{bmatrix} &= I^{-1}\begin{bmatrix} L \\ M \\ N \end{bmatrix} - I^{-1}\begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix} I \begin{bmatrix} P \\ Q \\ R \end{bmatrix}
\end{aligned} \tag{10}$$

A close examination of the above equations suggests that the above nonlinear equations of motion, which are typically written in the form, $\dot{x} = f(x) + g(x,\delta)$ can in fact be written as $\dot{x} = A(x)x + \mathbf{B}\delta$. This special form allows us the luxury of applying linear techniques to studying the robustness of the nonlinear system of equations, if we treat the above system as a '*State Dependent Linear State Space system*'.

Thus the State Dependent Linear state space system can be written as

$$\dot{x} = A(x)x + B\delta \tag{11}$$

In addition, the rotational motion equations possess some additional properties such as A(0) = 0 and

$$\begin{aligned}
&\dot{x} = [A'(x\ )]x + B\delta \\
&where \\
&A'(x) = \left[\frac{\partial A}{\partial x_1}x, \ldots\ldots\ldots, \frac{\partial A}{\partial x_n}x\right]
\end{aligned} \tag{12}$$

It may be noted that the satisfaction of the condition A(0) = 0 may be difficult in the event the state variable vector includes variables in addition to the angular velocities but the effect of that is negligible in practical situations in which the time scales of rotational angular velocities dominate the time scales of other state variables.

This in-turn enables us to utilize the property of point-wise controllability and stability to study the stability robustness of the nonlinear control system.
Now the generalized moments can be written as

$$\begin{aligned}
L &= L_{delta} + L_{BAE} \\
M &= M_{delta} + M_{BAE} \\
N &= N_{delta} + N_{BAE}
\end{aligned} \tag{13}$$

The state and control variables are:

$$x=\begin{bmatrix} P \\ Q \\ R \\ \phi \\ \theta \\ \psi \end{bmatrix} \quad and \quad \delta=\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} \tag{14}$$

The moments due to the control surface deflections can be written as

$$\begin{aligned} L_{delta} &= L_{\delta 1}\delta_1 + L_{\delta 2}\delta_2 + L_{\delta 3}\delta_3 + L_{\delta 4}\delta_4 \\ M_{delta} &= M_{\delta 1}\delta_1 + M_{\delta 2}\delta_2 + M_{\delta 3}\delta_3 + M_{\delta 4}\delta_4 \\ N_{delta} &= N_{\delta 1}\delta_1 + N_{\delta 2}\delta_2 + N_{\delta 3}\delta_3 + N_{\delta 4}\delta_4 \end{aligned} \tag{15}$$

The equations of motion are written in the following state space form

$$\begin{bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Wn(1,1) & Wn(1,2) & Wn(1,3) & 0 & 0 & 0 \\ Wn(2,1) & Wn(2,2) & Wn(2,3) & 0 & 0 & 0 \\ Wn(3,1) & Wn(3,2) & Wn(3,3) & 0 & 0 & 0 \\ 1 & \sin\phi\tan\theta & \cos\phi\tan\theta & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 & 0 & 0 \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \\ \phi \\ \theta \\ \psi \end{bmatrix} +$$

$$\begin{bmatrix} I^{-1}(1,1) & I^{-1}(1,2) & I^{-1}(1,3) \\ I^{-1}(2,1) & I^{-1}(2,2) & I^{-1}(2,3) \\ I^{-1}(3,1) & I^{-1}(3,2) & I^{-1}(3,3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L_{\delta 1} & L_{\delta 2} & L_{\delta 3} & L_{\delta 4} \\ M_{\delta 1} & M_{\delta 2} & M_{\delta 3} & M_{\delta 4} \\ N_{\delta 1} & N_{\delta 2} & N_{\delta 3} & N_{\delta 4} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} + \tag{16}$$

$$\begin{bmatrix} I^{-1}(1,1) & I^{-1}(1,2) & I^{-1}(1,3) \\ I^{-1}(2,1) & I^{-1}(2,2) & I^{-1}(2,3) \\ I^{-1}(3,1) & I^{-1}(3,2) & I^{-1}(3,3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L_{BAE} \\ M_{BAE} \\ N_{BAE} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where

Moments of Inertia Tensor

$$I=\begin{bmatrix} Jxx & 0 & -Jxz \\ 0 & Jyy & 0 \\ -Jxz & 0 & Jzz \end{bmatrix} \tag{17}$$

$$Wn=-I^{-1}\begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix} I \tag{18}$$

Thus

$$A(x)=\begin{bmatrix} Wn(1,1) & Wn(1,2) & Wn(1,3) & 0 & 0 & 0 \\ Wn(2,1) & Wn(2,2) & Wn(2,3) & 0 & 0 & 0 \\ Wn(3,1) & Wn(3,2) & Wn(3,3) & 0 & 0 & 0 \\ 1 & \sin\phi\tan\theta & \cos\phi\tan\theta & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 & 0 & 0 \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta & 0 & 0 & 0 \end{bmatrix} \tag{19}$$

$$B=\begin{bmatrix} I^{-1}(1,1) & I^{-1}(1,2) & I^{-1}(1,3) \\ I^{-1}(2,1) & I^{-1}(2,2) & I^{-1}(2,3) \\ I^{-1}(3,1) & I^{-1}(3,2) & I^{-1}(3,3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} L_{\delta 1} & L_{\delta 2} & L_{\delta 3} & L_{\delta 4} \\ M_{\delta 1} & M_{\delta 2} & M_{\delta 3} & M_{\delta 4} \\ N_{\delta 1} & N_{\delta 2} & N_{\delta 3} & N_{\delta 4} \end{bmatrix} \tag{20}$$

Closed Loop System under Dynamic Inversion

Let $\omega=\begin{bmatrix} P \\ Q \\ R \end{bmatrix}$ *and* $\omega = Cx$ (21)

Then

$$\begin{aligned} \dot{\omega}&=C\dot{x} \\ \dot{\omega}&=CA(x)x+CB\delta \end{aligned} \tag{22}$$

Then the dynamic inversion control law can be derived as

$$\delta=(CB)^{+}(\dot{\omega}_{des}-CA(x)x) \tag{23}$$

The closed loop system is given by

$$\dot{x} = A(x)x + B(CB)^{+}(\dot{\omega}_{des} - CA(x)x)$$
$$\dot{x} = [A(x) - B(CB)^{+}CA(x)]x + B(CB)^{+}\dot{\omega}_{des} \quad (24)$$

Let

$$\dot{\omega}_{des} = C_{des}.x \quad (25)$$

Then the closed loop system can be represented as

$$\dot{x} = [A(x) - B(CB)^{+}CA(x) + B(CB)^{+}C_{des}]x \quad (26)$$

In the above form of the closed loop system, proper choice of C_{des} will render the closed loop system stable.

Selection of Desired Dynamics via Linear Quadratic Regulator Approach

Note that the above closed loop system can be written in the form

$$\dot{x} = A(x) - BK_{gain} \quad (27)$$

where

$$K_{gain} = (CB)^{+}(CA(x) - (CB)^{+}C_{des} \quad (28)$$

The controllability of the pair $(A(x), B)$ is checked for every cycle and by selecting a proper gain K_{gain} during each cycle using linear techniques such as LQR, C_{des} can be calculated from Equation (28) to make the closed loop system in Equation (26) stable at all the points (of the state).
Recall that the Linear Quadratic Regulator method calculates the optimal gain matrix K_{gain} such that the state-feedback law u = -K_{gain}.x minimizes the cost function

$$J = \int (x'Qx + u'Ru)dt \quad (29)$$

subject to the state dynamics $\dot{x} = Ax + Bu$

Robustness Study for perturbations in the Control Derivatives

Having established a scheme to stabilize the closed loop systems for some 'nominal' parameter values, it is of interest to determine the robustness of this control scheme under parameter perturbations. The most critical parameters in the above dynamic inversion

based indirect adaptive control law are the control moment derivatives with respect to the control surface deflections. In order to study the stability robustness of the designed control system with respect to these control derivatives, the individual control derivatives are perturbed and the eigenvalues of the closed loop system are noted.
The closed loop system under perturbation can be represented as follows:

$$\dot{x}=[A(x)-B_{pert}(CB_o)^+CA(x)+B_{pert}(CB_o)^+C_{des}]x \tag{30}$$

where

$$B_{pert}=B_o+\Delta B \tag{31}$$

In the present exercise, we obtain the tolerable perturbations on these control derivatives by perturbing the control derivatives by a given amount and observing the closed loop system stability through the eigenvalue criterion. These simulation results are presented in the next section. As a future task, it is intended to study the robustness in a converse way i.e. given the allowable or tolerable perturbations in the individual control derivatives (in an interval), one can use the necessary and sufficient `extreme point' solution presented in [7] to test if the closed loop system is stable in the entire given ranges of the perturbations.

Application to a Reentry Vehicle and Simulation Results

Figure 2 shows the plots of P, Q, and R obtained by using the above method for the nominal values of the control derivatives. Figure 3 shows the plots of the corresponding control surface deflections.

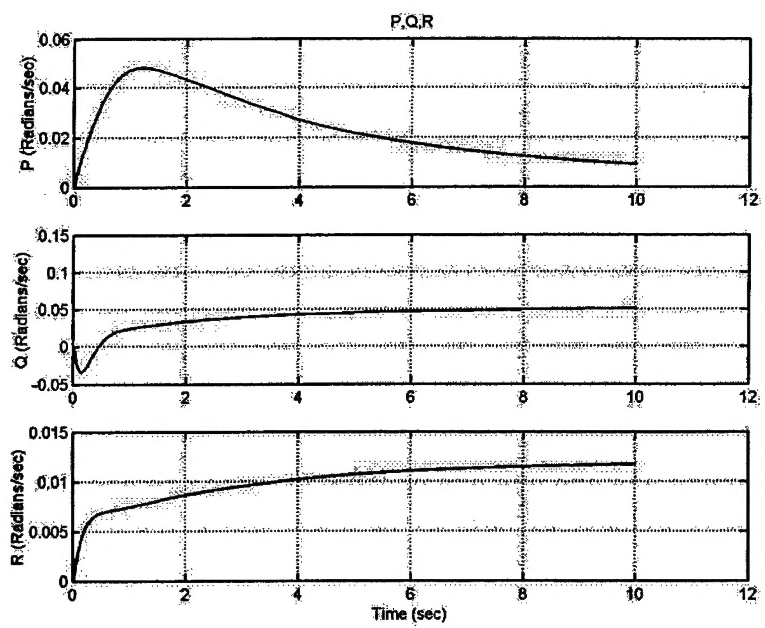

Figure 2

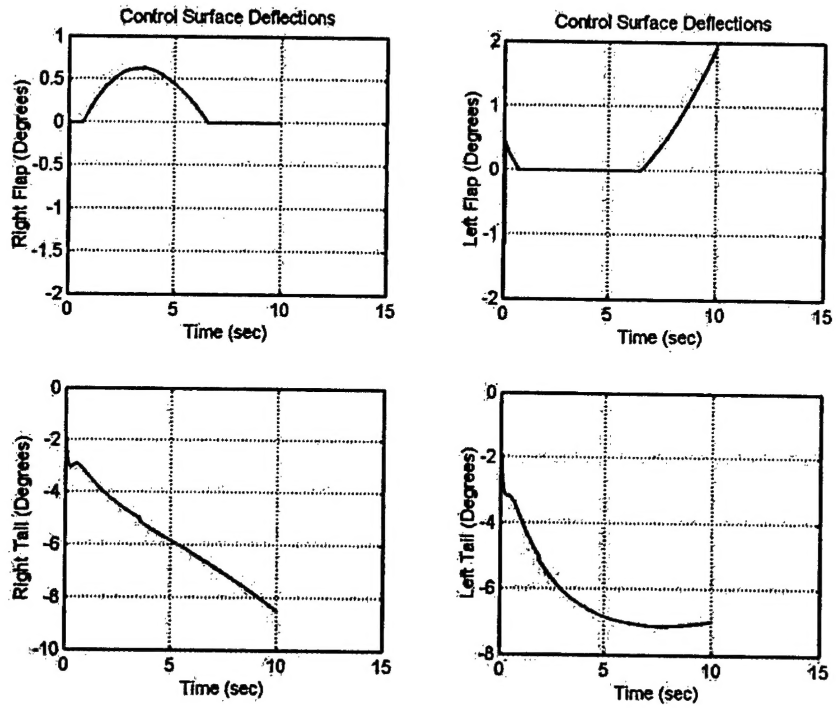

Figure 3

The results of control derivative perturbation are shown in Table 1. The closed loop system at each cycle is stabilized using LQR technique. Then the control derivatives are perturbed until the closed loop system became unstable. The table shows the extreme values of the control derivatives for which the closed loop system continued to be stable.

Control Derivative	Minimum Value	Maximum Value
$L_{\delta 1}$	-0.0038	+0.002
$L_{\delta 2}$	-0.0002	+0.0098
$L_{\delta 3}$	-0.0325	-0.0025
$L_{\delta 4}$	-0.0025	+0.0025
$M_{\delta 1}$	-0.9998	+0.0102
$M_{\delta 2}$	-0.0998	+0.2002
$M_{\delta 3}$	-0.0180	+0.0097
$M_{\delta 4}$	-0.0180	+0.0097
$N_{\delta 1}$	-0.0103	+0.0012
$N_{\delta 2}$	+0.0003	+0.0913
$N_{\delta 3}$	-0.0044	+0.0057
$N_{\delta 4}$	-0.0956	-0.0055

Conclusions

In this paper, a method is presented to determine the robustness of a baseline dynamic inversion control system developed for a representative space-maneuvering vehicle similar to the X-40A. In order to study the robustness of the controller, an innovative method that takes advantage of the specific nature of the equations of motions of the aircraft is used to aid in the analysis of the closed loop system. Using this technique, point-wise controllability and stability are shown to be sufficient to guarantee the stability of the overall closed loop system. The method exploits the unique nature of the equations of motion of the aircraft to develop a closed loop system under dynamic inversion so that appropriate perturbation study can be carried out on the system. The nonlinear equations of motion are expressed in the form that is termed State Dependent Linear State Space System and this allows us to carry out robustness studies on the nonlinear system using linear system techniques. Future research is directed towards including all the translational and rotational motion state variables into the model to better analyze the robustness of the control system.

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